Exam. Code : 211001
Subject Code : 4955

M.Sc. Mathematics 1st Semester (Batch 2021-23)

MECHANICS—I

Paper—MATH-554

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Derive the expressions of radial and transverse components of velocity and acceleration.

   (b) Prove that any rotation of a rigid body about a fixed point O is equivalent to a rotation about a definite axes through point O.

2. (a) Discuss general rigid body motion as a screw motion.

   (b) A car of width b is moving with constant velocity v close to the edge of a straight road. If a pedestrian steps on the road at a point distant d in front of the car, what is the least uniform velocity at which he must be able to walk in order to cross the road in safety ?

SECTION—B

3. (a) Show that a necessary and sufficient condition for a field of force $\bar{F}$ to be conservative is that curl $\bar{F} = 0$.

(b) Explain impulsive forces. An impulse $\bar{I}$ changes the velocity of the particle of mass m from $\bar{v}_1$ to $\bar{v}_2$. Show that the K.E. gained is $\frac{1}{2}\bar{I}(\bar{v}_1 + \bar{v}_2)$.

4. (a) A small stone of mass m is thrown vertically upwards with initial speed v. If the air resistance at speed v is mkv, show that the stone returns to its starting point with speed U given by :

$$g - kU = (g + kv)\exp\left[\frac{-k(U + v)}{g}\right]$$

(b) Find the velocity of the projectile at any point of its trajectory.

SECTION—C

5. (a) Prove that the orbit described by a central force is a plane curve.

(b) A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove, with the usual notation

$$v^2 = \mu(2/r - 1/a),\ h^2 = \mu a(1 - e^2)$$

when the particle is at one of a minor axis, its velocity is doubled. Prove that the new path is a hyperbola of eccentricity $(9 - 8e^2)^{1/2}$.

6. (a) State Kepler law of Planetary Motion. Two gravitating particles of masses m and M move under the force of their attraction. Show that the centre of mass of the two particles moves with constant velocity and that if $\bar{r}$ is the position vector of m relative to M then $\ddot{\bar{r}} = \frac{-\gamma(M + m)}{r^2}\hat{r}$ where γ is the gravitational constant.

(b) A particle P moves under a central force μr per unit mass along PO, O is the centre of force and $\overrightarrow{OP} = \bar{r}$. Discuss the orbit of the particle.

SECTION—D

7. (a) Define Moment of Inertia. State and prove theorem of parallel axes for moment of inertia.

(b) Derive angular momentum about a fixed a fixed point and about a fixed axes.

8. (a) Show that the necessary and sufficient condition for the system to be equimomental is that (i) they have same axes (ii) same centroid (iii) they have the same principal axes and principal moment of inertia at the centroid.

(b) Show that in two-dimensional mass distributions, the principal directions with usual notations are given by $\tan 2\alpha = \frac{2F}{B - A}$.